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Comment on "Josephson Current through a Nanoscale Magnetic Quantum Dot"

In a recent work [1], Siano and Egger (SE) studied the Josephson current through a quantum dot in the Kondo regime using the quantum Monte Carlo (QMC) method. Several of their results were unusual, and inconsistent with those from the numerical renormalization group (NRG) calculations[2, 3] among other previous studies. Those results in Ref.[1] are not reliable for the following two reasons: (i) The definition of the Kondo temperature was wrong; (ii) There were substantial finite-temperature effects.

We first clarify point (i). The *normal-state* Kondo temperature[4, 5] *in the absence of superconductivity* provides one of the most significant energy scales of the system. SE defined the Kondo temperature as

$$T_K^{\rm SE} = \exp[\pi\epsilon_0(\epsilon_0+U)/\Gamma_{\rm SE}U]\sqrt{\Gamma_{\rm SE}U}/2 \qquad (1)$$

with $\Gamma_{\rm SE} = 2\pi\rho_0|t|^2$, where $|t|^2$ denotes the coupling to *one lead* and ρ_0 the density of states (DOS) at the Fermi level. In Ref.[2] we defined it as

$$T_K = \exp[\pi\epsilon_0(\epsilon_0+U)/2\Gamma U]\sqrt{\Gamma U/2} \qquad (2)$$

with $\Gamma = 2\pi N_0|V|^2$, where $|V|^2$ denotes the coupling to *one lead* and N_0 the DOS at the Fermi level *per spin* [the factor 2 in the coupling comes from the two leads]. It is important to clarify the difference between the two definitions since different definitions of T_K result in significantly different scaling behaviors of physical quantities. We note that both forms, Eqs. (1) and (2), appear in the literature. However, in Eq. (1) $\Gamma_{\rm SE}$ should be the *full* width at half maximum of the single particle level of the noninteracting dot [6], whereas in Eq. (2) Γ should be the *half* width at half maximum (HWHM) of the single particle level. To see the precise meaning of $\Gamma_{\rm SE}$, let us take the limit $\Delta = 0$ and $U = 0$ in the local Green's function (GF) in Eq. (6) in Ref. [1]. Going over to the retarded GF, we find $G_R^{-1} \sim E + i\Gamma_{\rm SE}$, which yields the spectral function

$$A(E) = -\frac{1}{\pi}{\rm Im}G_R = \frac{1}{\pi}\frac{\Gamma_{\rm SE}}{E^2+\Gamma_{\rm SE}^2}\,. \qquad (3)$$

Therefore, $\Gamma_{\rm SE}$ corresponds to *HWHM*. Namely, the two hybridization $\Gamma_{\rm SE}$ and Γ are the same. Therefore, the two Kondo temperatures in Eqs. (1) and (2) are related with each other by $T_K^{\rm SE} = T_K^2/\sqrt{\Gamma U}$, which implies that the scale $\Delta/T_K^{\rm SE}$ differs from the scale given in our work [2]. The unusual definition of Kondo temperature in Eq. (1) explains the (otherwise) unusual behaviors of $I(\phi)$ with respect to U/Δ in Fig. 2 of SE.

We now move on to point (ii). In Ref. [1] all calculations have been done at a finite temperature $T = 0.1\Delta$ and SE note that "this appears to be quite close to the ground-state limit". This is particularly important in the determination of the current-phase relation. To estimate the Josephson energy we note that it is obtained from

$$E_J(\phi) = \int^{\phi} d\phi'\, I_S(\phi') \sim \Delta\frac{I_c}{I_c^{\rm short}}\,, \qquad (4)$$

where I_c is the effective critical current of the system and $I_c^{\rm short} \equiv e\Delta/\hbar$ the critical current of the open contact. According the the numerical results in Ref. [1], $I_c/I_c^{\rm short} \le 0.1$ for $\Delta/T_K^{\rm SE} \gtrsim 5$ ($\Delta/T_K \gtrsim 1$ in Ref. [2]). We think that in most plots in Ref. [1] the current-phase relation contains significant amount of thermal activation. To confirm this we have performed NRG calculations at finite temperatures and the results in Fig. 1 demonstrate the strong finite-temperature effects. The sharp transition at zero temperature is washed out and the critical current is reduced by a factor of 5 for $T/\Delta = 0.1$. The discrepancy between the NRG and QMC data in the new Fig. 2 of the Reply[7] may simply reflect the different estimates of critical value Δ_c/T_K (i.e., the NRG and QMC data are in different phases), and may not be an evidence that the NRG is less accurate.

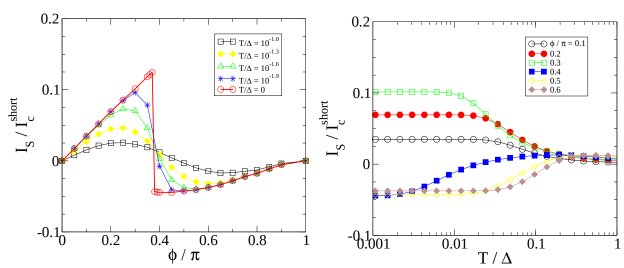

FIG. 1: (a) Josephson current $I_S(\phi)$ at different temperatures. (b) Josephson current as a function of temperature for different values of ϕ. $\Delta/T_K = 1.6$.

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